$H_0: \mu_1 = \dots = \mu_n$

18/5/17

	SS	DF	MS	F-test
	$SS_{tr} = \sum_{j=1}^{r} n_j(\bar{Y}_{\cdot j} - \bar{Y}_{\cdot\cdot})^2$ $= \sum_{j=1}^{r} \frac{Y_{\cdot j}^2}{n_j} - \frac{Y_{\cdot\cdot}^2}{n}$	$r-1$ 3	$MS_{tr} = \frac{SS_{tr}}{r-1}$ 8.41	$n=23$ $F = \frac{MS_{tr}}{MS_{res}}$
	$SS_{res} = SS_{tot} - SS_{tr}$	$n-r$ 19	$MS_{res} = \frac{SS_{res}}{n-r}$ 0.95	$F = \frac{8.41}{0.95}$
	$SS_{tot} = \sum_{j=1}^{r}\sum_{i=1}^{n_j}(Y_{ij} - \bar{Y}_{\cdot\cdot})^2$	22 $n-1$		

$F \geq F_{\alpha, r-1, n-r}$

$F \geq F_{0.05, 3, 14} (= 3.13)$

$E(X^2) = Var(X) + [E(X)]^2$

$n = \sum_{j=1}^{r} n_j$

$Y_{ij} = \mu_j + \varepsilon_{ij}, \quad j = 1, \dots, r, \quad i = 1, \dots, n \qquad Y_{ij} \sim N(\mu, \sigma^2)$

$E(MS_{tr}) = \sigma^2 + \frac{1}{r-1}\sum_{i=1}^{r}(\mu_j-\mu)^2$ / $\mu = \sum_{r=1}^{r} n_j\mu_j / n$ / Άσκηση 6.1

$$E(MS_{tr}) = E\left(\frac{SS_{tr}}{r-1}\right) = \frac{1}{r-1}\left[\sum_{j=1}^{r}\frac{E(Y_{\cdot j}^2)}{n_j} - \frac{E(Y_{\cdot\cdot}^2)}{n}\right]$$

$$E(Y_{\cdot j}) = \sum_{i=1}^{n_j} E(Y_{ij}) = \sum_{i=1}^{n_j}\mu_j = n_j\mu_j$$

$$Var(Y_{\cdot j}) = \sum_{i=1}^{n_j} Var(Y_{ij}) = \cancel{n_j\sigma^2} = \sum_{i=1}^{n_j}\sigma^2 = n_j\sigma^2$$

$E(Y_{\cdot j}^2) = n_j\sigma^2 + n_j^2\mu_j^2$

$$E(Y_{\cdot\cdot}) = \sum_{j}\sum_{i} E(Y_{ij}) = \sum_{j=1}^{r}\sum_{i=1}^{n_j}\mu_j = \sum_{j=1}^{r} n_j\mu_j = n\mu$$

$$Var(Y_{\cdot\cdot}) = \sum_{j}\sum_{i} Var(Y_{ij}) = \sum_{j}\sum_{i}\sigma^2 = n\sigma^2$$

$$\Rightarrow E(Y_{\cdot\cdot}^2) = n\sigma^2 + n^2\mu^2$$

$E(Y_{\cdot j}^2) = n_j\sigma^2 + n_j^2\mu_j^2$ κ' $E(Y_{\cdot\cdot}^2) = n\sigma^2 + n^2\mu^2$

$$E(MS_{tr}) = \frac{1}{r-1}\left[\sum_{j=1}^{r} \frac{n_j\sigma^2 + n_j^2\mu_j^2}{n_j} - \frac{n\sigma^2 + n^2\mu^2}{n}\right]$$

$$= \frac{1}{r-1}\left[\sum_{j=1}^{r} \sigma^2 + \sum_{j=1}^{r} n_j\mu_j^2 - \sigma^2 - n\mu^2\right]$$

$$= \sigma^2 + \frac{1}{r-1}\left[\sum_{j=1}^{r} n_j\mu_j^2 - n\mu^2\right]$$

Άσκηση 6.3
(Δεδομένα στο βιβλίο)
Τα αποτελέσματα πάνω στον πίνακα!

$H_0: \mu_u - \mu_v = 0$ v $H_\alpha: \mu_u - \mu_v \neq 0$

$$t = \frac{\bar{Y}_{\cdot u} - \bar{Y}_{\cdot v}}{\sqrt{MS_{res}\left(\frac{1}{n_u} + \frac{1}{n_v}\right)}} \quad , \quad |t| \geq t_{\alpha/2, n-r}$$

Έλεγχος: $|\bar{Y}_{\cdot u} - \bar{Y}_{\cdot v}| \geq t_{\alpha/2, n-r} \cdot \sqrt{MS_{res}\left(\frac{1}{n_u} + \frac{1}{n_v}\right)}$

$H_0: \mu_4 - \mu_3 = 0$ v $H_\alpha: \mu_4 - \mu_3 > 0$ $\alpha = 0.05$

$\bar{Y}_{\cdot 3} =$, $\bar{Y}_{\cdot 4} =$, $n_3 = 6$, $n_4 = 4$ $t_{0.05, 19} = 1.729$

$$t = \frac{\bar{Y}_{\cdot 4} - \bar{Y}_{\cdot 3}}{\sqrt{MSE\left(\frac{1}{n_3} + \frac{1}{n_4}\right)}} = \frac{9-6}{\sqrt{0.48\left(\frac{1}{6}+\frac{1}{4}\right)}} = 4.7721221$$

[illegible]

Άσκηση 3.20

$f(x) = \frac{1}{x^2}$, $x \geq 1$ $n = 72$, $X_1, \dots, X_{72}$ τδ.

P(περ από 50 ≤ 3) =

$E = \{$ η παρατηρ. τιμή ≤ 3 $\}$, $p = P(E) = \int_1^3 f(x)dx = -\frac{1}{x}\Big|_1^3 = \frac{2}{3}$

Y = αριθμός τιμών του τδ που είναι ≤ 3
= αριθμός E $n = 72$ δοκιμές με $p = P(E)$

$Y \sim B(n = 72, p = \frac{2}{3}) \rightarrow P(Y > 50) = P(Y \geq 51) =$

$\rightarrow P(Y > 50) = \sum_{x=51}^{72} \binom{72}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{n-x} = \dots$

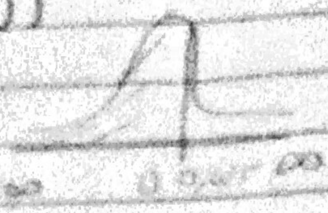
$Y \sim B(n = 72, p = \frac{2}{3}) \overset{\text{περ}}{\sim} N(np = 72 \times \frac{2}{3} = 48, npq = 72 \times \frac{2}{3} \times \frac{1}{3} = 16)$

$\rightarrow$ P(περ από 50 ≤ 3) $= P(Y > 50) = P(Y \geq 51 | Y \sim B(72, \frac{2}{3})) \overset{\text{προσ}}{\approx}$
$P(Y \geq 50.5 | Y \overset{\text{περ}}{\sim} N(48, 16)) = P\left(\frac{Y - \mu}{\sigma} (=Z) \geq \frac{50.5 - 48}{\sqrt{16}} \Big| Z \sim N(0,1)\right)$

$= P(Z \geq 0.625) = 0.27$

$P(Y > 50) = 1 - P(0 \leq Y \leq 50) = 1 - \cancel{P(-0.5 \leq Y \leq 50.5 |}$ προσ
$= 1 - P(-0.5 \leq Y \leq 50.5 | Y \overset{\text{περ}}{\sim} N(72, 16))$

$= 1 - P(-1.2125 \le Z \le 0.625 \mid Z \sim N(0,1))$



$= 1 - (0.5 + 0.23) = 0.27$

Άσκηση για επανάληψη (17)

6, 7, 9, 8, 10, 8, 11, 7 $\quad n = 16 \quad N(\mu, \sigma^2)$

6, 5, 10, 5, 11, 9, 10, 6

$\alpha = 0.05$

i) Καλο-γραφ; όταν $\mu = 10$ κ' 95% ΔΕ για μ

(ii) Αν $\sigma^2 = 9$ το (i) κ' ισχύ για $H_\alpha: \mu = 9$

Λύση

(i) $H_0: \mu \geq 10$ v $H_\alpha: \mu < 10$

$\bar{x} = 8$, $s^2 = 4.26$ $(s = 2.064)$

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ και κρ. περιοχή $t \le -t_{\alpha, n-1}$

$(-t_{0.05, 15} = -1.753)$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t = \frac{8 - 10}{2.064/\sqrt{16}} = -3.88$$

Επειδή $-3.88 < -1.753$ απορ. H_0

(ii) $H_0: \mu \geq 10$ v $H_a: \mu < 10$

$\bar{x} = 8$, $a = 0.05$, $\sigma^2 = 9$

$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \to Z = \frac{8-10}{3/\sqrt{16}} = -2.67$ $\left(Z < -z_a = -z_{0.05} = -1.645 \right)$

Επειδή $-2.67 < -1.645$ απορ. H_0

(2.13)

$(1-a)\cdot 100\%$ Δ.Ε. για μ: $\bar{x} \pm t_{0.025}\frac{S}{\sqrt{n}} \to [9.411, 9.081]$

$\gamma = 1-\beta$

$\beta = P(\text{δεχ. } H_0 \mid H_a \text{ αληθ.})$

$= P\left(\frac{\bar{X}-\cancel{10}}{\sigma/\sqrt{n}} \geq -1.645 \mid \mu = 9 \right)$

$\frac{\bar{X}-9}{\sigma/\sqrt{n}} \sim N(0,1)$

Άρα $\beta = P\left(\frac{\bar{X}-9}{\sigma/\sqrt{n}} \geq -1.645 + \frac{1}{\sigma/\sqrt{n}} \mid \frac{\bar{X}-9}{\sigma/\sqrt{n}} \sim N(0,1) \right)$

$= P\left(Z \geq -1.645 + \frac{4}{3} \mid Z \sim N(0,1) \right)$

$= P(Z \geq -0.312) = 0.5 + 0.1225$

κ' $\gamma = 1 - \beta = 0.6225$